

never round up
 for a polygon
 it will leave the
 solution set

STEPS:
 graph, test
 and shade, use
 (0,0) and if it goes
 through origin use

Symbols: < - less than > - greater than
 <= - less than or equal to (max) ≥ - greater than or equal to (min)

4 types of sentences:
Simple total: a situation where there is a total of two objects (x, y) ex. $x+y=80$ **Complex total:** a situation where there is a total, but its different from although related to the variables ex. A class of boy and girls raise money. each boy brings in \$2, and each girl brings \$3, The class raises \$60. $2x+3y=60$ **Unique face:** only contains one variable ex. $x=5$ $y=2$ **Compare 2 variables:** no total, will equate the # of a variable to the # of the other ex. $x=y+4$, $x=2y$, $y=4x$ **Non-unique optimal solution:**

Comparison method: isolate "y" in equations more than 1 vertex can optimize to, (0,0) ex. $2x < y$ equate x chunks, solve for x, sub x to find y. a tie, all points between vertices will be true shade towards

Steps: find (x, y), formulate constraints, graph, pick best ones, T0: goal of problem, define by rule, \$ based (\$)

$P = R - C$ (want to max R + P and min C) MAKE SURE POINT F IS IN SOLUTION SET?

Julie's Proposal: charge for car: \$5 charge of van: \$8

$P = 5x + 8y$ $A(0, 150) = 1200$
 $B(150, 150) = 1950$ $C(300, 0) = 1500$
 $D(90, 0) = 450$ $E(0, 90) = 720$

Order: * include conditions, max or min, usually a trip, **Roger's Proposal:** charge car: \$7, charge van: \$10

max of 210 vehicles, max 150 vans, at least 90 vehicles, at most twice as many cars as vans

$P = 7x + 10y$ $x + y \leq 210$, $y \leq 150$
 $x \geq 90$, $x \leq 210$

vertices: points **Optimal chain / Cycle:** method is trial and error (3) Which is the best? May sometimes

order: * include conditions, max or min, usually a trip, **journey:** works from a network

adjacent: connected **Optimal Tree:** * avoid unnecessary * by an edge. Key words: pipes, wires, clearing, want everything

connected: can get connected, fewest edges ($n-1$) Method: GROW to every point COPY original, DON'T FORGET TO WRITE

complete: can get MIN TREE and list, pick best edges, stop to any vertex from every when connected. TREES MUST BE CONNECTED

vertex: AND NO SIMPLE CYCLE?

chain/path: edges **CRITICAL PATHS:** leaving and arriving represents minimum time to complete at different vertex 3 types: read a path = name paths, get length: # of edges values, pick critical ones, Build + Read: use minimum # of colors, travelled distance: use table to build, then read.

length of shortest path: Change it: read or make original, of highest degree, complete triangle.

simple C/P: make change, read, analyze difference

not repeat edges

cycles/loops: edges begin and end at same vertex

example of adjacent:

$\text{speed} = \frac{\text{distance}}{\text{time}}$ $\text{distance} = \text{speed} \cdot \text{time}$ $\text{time} = \frac{\text{distance}}{\text{speed}}$ $\text{speed} = \frac{\text{distance}}{\text{time}}$

Diagram:

Systems of Equations

$$Y = -2x - 3 \quad 5x + y = 3 \\ Y = 5x - 5x$$

$$-2x - 3 = -3 - 5x \\ -2x + 5x = -3 + 3 \\ 3x = 0 \quad x = 0$$

Graphing

$$5x + y = 0 \quad -2x - 3 = 0 \\ y = -5x \quad 2x = 3 \\ y = 5x \quad x = \frac{3}{2}$$

Corner Points

Target Conversion

$$P = R - C$$

Max Min

- ① Identify x, y
- ② Formulate constraints
- ③ Graph, test, shade
- ④ Corner Points

P = R - C	Total
A	B

Simple Total

$$X + Y = 30$$

X	Y
0	30
30	0

$$0 + Y = 30 \\ Y = 30$$

$$X + 0 = 30 \\ X = 30$$

Complete Total

$$2x + 3y = 60$$

$$\frac{2x}{2} + \frac{3y}{3} = \frac{60}{2} \\ x + y = 20$$

*** True: Shade towards tested pts
* False: away from tested pts**

Graph

Connected: Each vertex is connected to every other by one or more edges

Complete: Each pair of vertices are directly connected (Complete graph has every vertex with a degree = order - 1)

Chains/Path: A sequence of edges leaving one vertex and arriving at another

Length: Refers to the # of edges travelled in a path

Distance: The length of the shortest path between 2 vertices

Simple Chain/Path: A chain that does not repeat edges.

Cycles/Circuits: A sequence of edges beginning and ending at the same vertex (Chain goes back to it)

Simple Cycles/Circuits: Cycles that don't repeat edges

Euler chains/paths: A simple chain that touches every edge

Euler cycle/circuit: A simple cycle that touches every edge exactly once

Rule: All vertices must be of even degree

Hamiltonian Path/chain: A path that touches every vertex exactly once in a graph

Hamiltonian Cycle/Circuit: A cycle that touches every vertex once except the origin which it will end at

adjacent: Vertices connected by edge

Incident: A line connecting 2 dots

Tree

For must be connected
No simple cycle

Ex.: # of edges needed until bc order - 1 ($n-1$)

- A tree connects a graph with fewest $\frac{n(n-1)}{2}$ of edges
- Key words: villages not tree
 - possible roads
 - Start from main vertex

Plato's $\rightarrow 56$

10	15	20	5	4	2
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3	D	A	C	B	A	D
2	A	B	A	C	C	B
3	B	C	B	A	B	C

(A=12) \rightarrow winner
B=83
C=95
D=36

Never loses
 \rightarrow NEVER loses

10	14	16	10
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1 A C B A
2 B A A C
3 C B C B

A vs B \rightarrow A wins
AVL \rightarrow A wins
BVC \rightarrow B wins

A wins all matches!

Compare 2 variables

$$x = 6y \quad x | y$$

Let $y = 2$

0	0
12	2

$$x = 12$$

**True: Shade towards tested pts
False: away from tested pts**

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Directed Graph

↳ One way edges

Majority Rule:

- First place votes: $V = 50\% \text{ of total}$

Plurality: 1st place vote

Winning

10	10	9	11	11	11
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Elimination (look for majority 1st place votes)

10	12	8	6	9
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$10 + 12 = 22 \rightarrow 3$

Final $A = 16 + 8 = 24$
 $B = 12$ (eliminated)
 $C = 9 + 8 = 17 + 12 = 29$
 $D = 8$ (eliminated)

Optimal Choice

- Best one
- Usually trip, Journey method: trial & error
- Always try 3 method

Weighted Graphs

Length: How many lines

Value: Total of # of lines (Add)

Euler Examples

= path(1236254)
= cycle(345635261)
= simple path connecting (R, Q) \rightarrow RSPTQ

Simple circuit: min
Distance: $(P, Q) = 3$
Simple path connecting (R, Q) \rightarrow RSPTQ
Length of path: MHSNQ: 5

CRITICAL POINT

D Name left to right

SABDE Done: 22

Always the highest #

Name all possible routes

D Build

Change it

Analyze difference



Reflection

Sx
X axis
(flip)

Sy
Y axis
(flip)

$S\Box$ Pos
 $\begin{array}{|c|c|} \hline x+y & \\ \hline \end{array}$

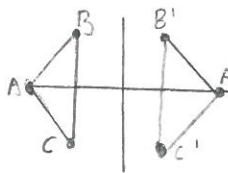
$S\Box$ Neg
 $\begin{array}{|c|c|} \hline x-y & \\ \hline \end{array}$

$Sx: (x,y) \rightarrow (x,-y)$

$Sy: (x,y) \rightarrow (-x,y)$

$S\Box: (x,y) \rightarrow (y,x)$

$\Box: (x,y) \rightarrow (-y,-x)$



* negative means change sign +, -

Rotation

$+ \rightarrow r = 270 \rightarrow r 90 = (x,y) \rightarrow (-y,x)$

$- \rightarrow r = 180 \rightarrow r 180 = (x,y) \rightarrow (-x,-y)$

$- \rightarrow r = 90 \rightarrow r 270 = (x,y) \rightarrow (y,-x)$

Geometry * 3D shapes: equivalent = same volume
2D shapes: equivalent = same area
Area: U^2 Volume: U^3

$P = 4 \times s$

$A = s^2$

$P = 2L + 2W$

$A = L \times W$

$C = 2\pi r$

$A = \pi r^2$

Triangles

$P = s_1 + s_2 + s_3$

$A = \frac{b \cdot h}{2}$

$P = s_1 + s_2 + s_3 + s_4$

$A = \frac{B_1 + B_2 \cdot h}{2}$

$P = p \cdot \text{perimeter}$

$A = \frac{p \cdot r}{2}$

$A = b \cdot h$

$P = b_1 + b_2 + b_3 + b_4$

$A = \frac{a_1 + a_2 \cdot h}{2}$

$A = \frac{d \cdot D}{2}$

$P = p \cdot \text{perimeter}$

$A = \frac{a \cdot c \cdot \sin B}{2}$

$V = \pi r^2 \cdot h$

$L_A = \text{Circumference} \cdot h$

$T_A = L_A + Bases$

$V = \pi r^2 \cdot h$

$L_A = 2\pi r \cdot h$

$T_A = 2\pi r h + 2\pi r^2$

$V = L \cdot w \cdot h$

$L_A = \text{Perimeter} \cdot h$

$T_A = L_A + \text{Bases}$

$V = s^3$

$L_A = 4s \cdot s$

$T_A = 4s^2 + 2s^2 = 6s^2$

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

$L_A = 4s \cdot s$

$T_A = 4s^2 + 2s^2 = 6s^2$

How to find r with:

Volume

$V = 523.6$

$D) r = \frac{4\pi r^3}{3}$

$2) 3V = 4\pi r^3$

$3) \frac{3V}{4\pi} = r^3$

$4) 3\sqrt{\frac{3V}{4\pi}} = r$

$5) 3\sqrt{\frac{3(523.6)}{4\pi}}$

$= 3\sqrt{1.25} = 5 = r$

Area

$A = 314.15$

$A = 4\pi r^2$

$2) 2\sqrt{\frac{A}{4\pi}}$

$3) 2\sqrt{\frac{314.15}{4\pi}}$

$4) 2\sqrt{25}$

$= 5 = r$

How to find Apothem

1) Central angle: $360^\circ : n$

2) Drop \perp (ut. in half)

$a = \frac{s}{n}$

$a = \frac{5}{12} = \frac{5}{12} \cdot \frac{360}{36} = \frac{5}{12} \cdot 10 = 6.25$

$a = 6.25$

$r = 12$

$r = 12$